MEM6804 Modeling and Simulation for Logistics and Supply Chain: Theory & Analysis

Sino-US Global Logistics Institute
Shanghai Jiao Tong University

Spring 2021 (full-time)

Assignment 3

Due Date: May 12 (in class)

Instruction

(a) You can answer in English or Chinese or both.
(b) Show enough intermediate steps.
(c) Write your answers **INDEPENDENTLY**.

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Question 1 $(2 + 8 = 10$ points)

If a machine produces some products one by one. The time length of producing one piece of product follows $\mathrm{Exp}(a)$. The finished products are stored in a warehouse, whose capacity limit is b pieces of products. When the warehouse is full, the machine will pause; once there is available space in the warehouse, the machine will continue the previous job immediately. Customers arrive to the warehouse following a Poisson process with rate c. Each customer will take away one piece of product (ignoring the time length of picking the product). If an arriving customer finds the warehouse is empty, he/she leaves immediately.

(1) If we are interested in the number of products in the warehouse, can this problem be represented by one of the queueing models introduced in Lec 3? (Only need to answer Yes or No.)
(2) If Yes, write down the queueing model and the corresponding parameters, and explain the reason; If No, add or modify some assumptions of this problem so that it can be represented by one of the queueing models introduced in Lec 3, and write down the queueing model and the corresponding parameters.

Question 2 $(3 + 4 + 3 + 5 + 5 = 20$ points)

Compute L, W, L_Q, W_Q for the following three queueing models:

(1) $M/M/1$, $\lambda = 0.6$, $\mu = 1$.
(2) $M/M/2$, $\lambda = 0.6$, $\mu = 0.5$.
(3) $M/G/1$, $\lambda = 0.6$, service time follows $\mathrm{Unif}(0, 2)$.

Based on the above results, answer the following two questions:

(4) Compare models (1) and (2), which one has higher efficiency (in terms of customer flow)? Based on which quantity? Provide an *intuitive explanation* for such difference.

(5) Compare models (1) and (3), which one has higher efficiency (in terms of customer flow)? Based on which quantity? Provide an *intuitive explanation* for such difference.

Question 3 $(4+4+2=10$ points)

Consider an $M/M/1/5$ queue with arrival rate $\lambda = 10$/hour and service rate $\mu = 8$/hour.

(1) What is the probability that an arriving customer finds the station is full?

(2) What is the expected amount of time a customer who enters the station will spend in it?

(3) What is the expected amount of time an arriving customer will spend in the station?

Question 4 (10 points)

Consider a Jackson queueing network with external arrival rate $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \lambda_3]^\intercal = [6, 2, 4]^\intercal$, service rate $\boldsymbol{\mu} = [\mu_1, \mu_2, \mu_3]^\intercal = [6, 4, 12]^\intercal$, server number $\boldsymbol{s} = [s_1, s_2, s_3]^\intercal = [2, 3, 1]^\intercal$, and routing matrix

$$\boldsymbol{P} = \begin{bmatrix} 0 & 0.6 & 0.2 \\ 0 & 0 & 0.4 \\ 0 & 0.5 & 0.1 \end{bmatrix}.$$

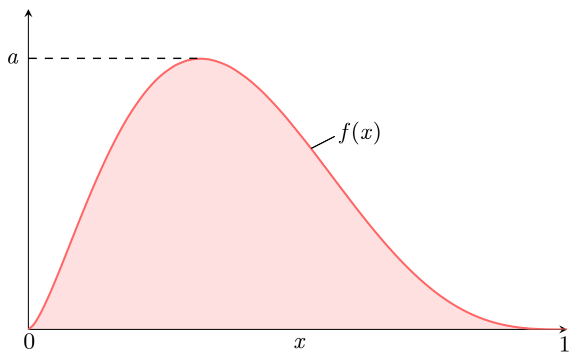
Calculte the expected number of customers in the entire network in steady state.

Question 5 (5 points)

$X \sim$ Weibull (α, β), if the pdf is $f(x) = \alpha\beta^{-\alpha}x^{\alpha-1}e^{-(x/\beta)^\alpha},\ x > 0, \alpha > 0, \beta > 0$. Use the inverse-transform technique to generate random variates from Weibull $(1/2, 1/4)$. (Use random numbers 0.8147, 0.9058, 0.1270, 0.9134 to generate 4 random variates; keep 4 decimals.)

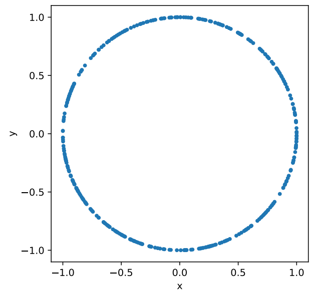
Question 6 (5 points)

Suppose $f(x)$, $0 \le x \le 1$, is the pdf of a random variable X, and it is as shown in the following figure. Suppose we can run `RANDX()` function to get the random variates from X. How to use `RANDX()` to get randomly and uniformly distributed points in the red area?



Question 7 (20 points)

Design **two** methods to randomly and uniformly sample points on the unit circle, as shown in the following figure. And prove that the methods are valid.



Question 8 (20 points)

Design **two** methods to randomly and uniformly sample points on the unit sphere (仅在球面上, 不含球体内部), as shown in the following figure. And prove that the methods are valid.